

Form factors, branching ratio and forward-backward asymmetry in $B \rightarrow K_1 \ell^+ \ell^-$ decays

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Abstract

We study long-distance effects in rare exclusive semileptonic decays $B \rightarrow K_1 \ell^+ \ell^-$, K_1 is the axial vector meson. The form factors, describing the meson transition amplitudes of the effective Hamiltonian, are calculated using Ward identities which are then used to calculate branching ratio and forward-backward asymmetry in these decay modes. The zero of forward-backward asymmetry is of special interest and provide us the precision test of Standard model.

1 Introduction

The investigation of rare semileptonic decays of the B meson induced by the flavor-changing-neutral-current (FCNC) transitions $b \rightarrow s$ provide potentially stringent tests of standard model (SM) in flavor physics. In SM these FCNC transitions are not allowed at tree level but are induced by the Glashow-Iliopoulos-Miani (GIM) amplitudes [1] at the loop level. Additionally these are also suppressed in SM due to their dependence on the weak

mixing angles of the quark-flavor rotation matrix – the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. These two circumstances make the FCNC decays relatively rare and hence important for the presence of new physics, commonly known as physics beyond SM.

The experimental observation of inclusive [3] and exclusive [4] decays, $B \rightarrow X_s \gamma$ and $B \rightarrow K^* \gamma$, has prompted a lot of theoretical interest on rare B meson decays. However, in case of exclusive decays any reliable extraction of the perturbative (short-distance) effects encoded in the Wilson coefficients of the effective Hamiltonian [5, 6, 7, 8, 9] requires an accurate separation of the nonperturbative (long-distance contributions), which therefore should be known with high accuracy. The theoretical investigation of these contributions encounters the problem of describing hadron structure, which provides the main uncertainty in the predictions of exclusive rare decays. In exclusive $B \rightarrow K, K^*$ decays the long-distance effects in the meson transition amplitude of the effective Hamiltonian are encoded in the meson transition form factors. Many exclusive $B \rightarrow K (K^*) \ell^+ \ell^-$ [10, 11, 12], $B \rightarrow \gamma \ell^+ \ell^-$ [13], $B \rightarrow \ell^+ \ell^-$ [14] processes based on $b \rightarrow s (d) \ell^+ \ell^-$ have been studied in literature and many frameworks have been applied to the description of meson transition form factors: among them the worth mentioning are constituent quark models, QCD sum rules, lattice QCD, approaches based on heavy quark symmetry and analytical constraints. Many observables like Forward Backward (FB) asymmetry, single and double lepton polarization asymmetries associated with the final state leptons, have been extensively studied for quite some time for quark level processes $b \rightarrow s (d) \ell^+ \ell^-$.

Recently, Belle[15] has announced the first measurement of $B \rightarrow K_1^+(1270) \gamma$

$$\mathcal{B}(B^+ \rightarrow K_1^+ \gamma) = (4.28 \pm 0.94 \pm 0.43) \times 10^{-5}. \quad (1)$$

after which these radiative decays became topic of prime interest and their is lot of theoretical progress for which one can see the Refs.[16, 17]. In this paper we study the semileptonic B meson decay $B \rightarrow K_1 \ell^+ \ell^-$ using the framework of Gilani *et al.*[18] with K_1 is an axial vector meson. The axial vector mesons is distinguished by vector by an extra γ_5 in the gamma structure of decay amplitude (DA) and some non perturbative parameters. But the presence of extra γ_5 does not alter the calculation except the switching of vector to axial vector form factors and vice a versa. As mentioned earlier, the theoretical understanding of exclusive decays is complicated mainly due to non-perturbative form factors entered in the long distance non-perturbative

contributions. The aim of this work is to relate the various form factors in model independent way through Ward identities. This enables us to make a clear separation between non-pole and pole type contributions, the $q^2 \rightarrow 0$ behavior of the former is known in terms of a universal function $\xi_\perp(0) \equiv g_+(0)$ introduced in the large energy effective theory (LEET) of heavy (B) to light (K_1) form factors[17]. The residue of the pole is then determined in a self consistent way in terms of $g_+(0)$ or $\xi_\perp(0)$ which will give information about the couplings of $B^*(1^-)$ and $B_A^*(1^+)$ with BK_1 channel. The form factors are then determined in terms of the known parameters like $g_+(0)$ and the masses of the particles involved which are then used to calculate the branching ratio and forward-backward asymmetry for these decays.

This paper is organized as follows: In section II we introduce the effective Hamiltonian formalism of semileptonic B meson decays and will write down the matrix elements for $B \rightarrow K_1 \ell^+ \ell^-$ decays. Section III discusses the Ward identities and develop the relationship between form factors which results in the reduction of number of unknown quantities. The form factors thus obtained are used for the calculation of decay width and forward-backward asymmetry. Finally, in the last section we summarize our conclusions.

2 Effective Hamiltonian

At quark level the decay $B \rightarrow K_1 \ell^+ \ell^-$ is similar to one studied in, for example, reference [10]. The basic transition $b \rightarrow s \ell^+ \ell^-$ is described by the effective Hamiltonian given below

$$H_{eff} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \quad (2)$$

where O_i 's are four local quark operators and \dot{C}_i are Wilson coefficients calculated in Naive dimensional regularization (NDR) scheme [19].

One can write the above Hamiltonian in the following free quark decay amplitude

$$\mathcal{M}(b \rightarrow s \ell^+ \ell^-) = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left\{ \begin{array}{l} C_9^{eff} [\bar{s} \gamma_\mu L b] [\bar{\ell} \gamma^\mu \ell] \\ + C_{10} [\bar{s} \gamma_\mu L b] [\bar{\ell} \gamma^\mu \gamma^5 \ell] \\ - 2 \hat{m}_b C_7^{eff} [\bar{s} i \sigma_{\mu\nu} \frac{\hat{q}^\nu}{\hat{s}} R b] [\bar{\ell} \gamma^\mu \ell] \end{array} \right\} \quad (3)$$

with $L/R \equiv \frac{(1 \mp \gamma_5)}{2}$, $s = q^2$ which is just the momentum transfer from heavy to light meson. The amplitude given in Eq.(3) is a free quark decay amplitude which contains certain long distance effect from the matrix element of local quark operators, $\langle l^+ l^- s | O_i | b \rangle$, $1 \leq i \leq 6$, which usually reabsorbed into the redefinition of short distance Wilson coefficients. Specifically, for the exclusive decays, the effective coefficients of the operator $O_9 = \frac{e^2}{16\pi^2} (\bar{s} \gamma_\mu L b) (\bar{l} \gamma^\mu l)$ can be written as

$$C_9^{eff} = C_9 + Y(\hat{s}) \quad (4)$$

where the perturbatively calculated result of $Y(\hat{s})$ is [19, 20]

$$Y_{\text{pert}}(\hat{s}) = \begin{aligned} & g(\hat{m}_c, \hat{s}) (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ & - \frac{1}{2} g(1, \hat{s}) (4C_3 + 4C_4 + 3C_5 + C_6) \\ & - \frac{1}{2} g(0, \hat{s}) (C_3 + 3C_4) + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6). \end{aligned} \quad (5)$$

For the values of the Wilson coefficients and the explicit expressions of g 's appearing in Eq. (5) we will refer to [19, 20]. The hat denotes normalization in terms of the B meson mass[10].

3 Matix Elements and Ward Identities

Exclusive decays $B \rightarrow K_1 \ell^+ \ell^-$ involve the hadronic matrix elements of the quark operators in Eq. (3) between B and K_1 . These can be parameterized in terms of form factors which are the scalar functions of the four momentum square ($q^2 = (p_B - p_{K_1})^2$). For the process we are considering, there are seven form factors like the transition of psudoscalar to vector meson. The non-vanishing matrix elements are

$$\begin{aligned} \langle K_1(k, \varepsilon) | V_\mu | B(p) \rangle &= i\varepsilon_\mu^* (M_B + M_{K_1}) V_1(s) \\ &\quad - (p + k)_\mu (\varepsilon^* \cdot q) \frac{V_2(s)}{M_B + M_{K_1}} \\ &\quad - q_\mu (\varepsilon \cdot q) \frac{2M_{K_1}}{s} [V_3(s) - V_0(s)] \end{aligned} \quad (6)$$

$$\langle K_1(k, \varepsilon) | A_\mu | B(p) \rangle = \frac{2i\varepsilon_{\mu\nu\alpha\beta}}{M_B + M_{K_1}} \varepsilon^{*\nu} p^\alpha k^\beta A(s) \quad (7)$$

with $V_\mu = \bar{s} \gamma_\mu b$ and $A_\mu = \bar{s} \gamma_\mu \gamma_5 b$ are the vector and axial vector currents respectively and ε_μ^* is the polarization vector for the final state axial vector meson. In Eq.(6)

$$V_3(s) = \frac{M_B + M_{K_1}}{2M_{K_1}} V_1(s) - \frac{M_B - M_{K_1}}{2M_{K_1}} V_2(s) \quad (8)$$

with

$$V_3(0) = V_0(0).$$

In addition to the above form factors there are also some penguin form factors which are:

$$\begin{aligned} \langle K_1(k, \varepsilon) | \bar{s} i \sigma_{\mu\nu} q^\nu b | B(p) \rangle &= [(M_B^2 - M_{K_1}^2) \varepsilon_\mu - (\varepsilon \cdot q)(p + k)_\mu] F_2(s) \\ &+ (\varepsilon^* \cdot q) \left[q_\mu - \frac{s}{M_B^2 - M_{K_1}^2} (p + k)_\mu \right] F_3(s) \end{aligned} \quad (9)$$

$$\langle K_1(k, \varepsilon) | \bar{s} i \sigma_{\mu\nu} q^\nu \gamma_5 b | B(p) \rangle = -i \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha k^\beta F_1(s) \quad (10)$$

with

$$F_1(0) = 2F_2(0).$$

The various form factors appearing in Eqs. (6)-(10) can be related by Ward identities as follows [18, 22, 23]

$$\langle K_1(k, \varepsilon) | \bar{s} i \sigma_{\mu\nu} q^\nu b | B(p) \rangle = -(m_b + m_s) \langle K_1(k, \varepsilon) | \bar{s} \gamma_\mu b | B(p) \rangle \quad (11)$$

$$\begin{aligned} \langle K_1(k, \varepsilon) | \bar{s} i \sigma_{\mu\nu} q^\nu \gamma_5 b | B(p) \rangle &= (m_b - m_s) \langle K_1(k, \varepsilon) | \bar{s} \gamma_\mu \gamma_5 b | B(p) \rangle \\ &+ (p + k)_\mu \langle K_1(k, \varepsilon) | \bar{s} \gamma_5 b | B(p) \rangle. \end{aligned} \quad (12)$$

Now we make the heavy quark approximation and compare coefficients of ε_μ^* and q_μ from both sides. In the heavy quark approximation we need not to compare the coefficients $(p + k)_\mu$. Using Eqs. (6)-(10) in Eqs. (11) and (12), we get the following relationship between form factors

$$F_1(s) = -\frac{(m_b - m_s)}{M_B + M_{K_1}} 2A(s) \quad (13)$$

$$F_2(s) = -\frac{(m_b + m_s)}{M_B - M_{K_1}} V_1(s) \quad (14)$$

$$F_3(s) = \frac{2M_{K_1}}{s} (m_b + m_s) [V_3(s) - V_0(s)]. \quad (15)$$

These are model independent results derived by using Ward identities. The universal normalization of the above form factors at $q^2 = s = 0$ is obtained by defining [18]

$$\begin{aligned} \langle K_1(k, \varepsilon) | \bar{s} i \sigma^{\alpha\beta} \gamma^5 b | B(p) \rangle &= -i \epsilon^{\alpha\beta\rho\sigma} \varepsilon_\rho^* [(p + k)_\rho g_+ + q_\sigma g_-] - (q \cdot \varepsilon^*) \epsilon^{\alpha\beta\rho\sigma} (p + k)_\rho q_\sigma h \\ &- i [q^\alpha \epsilon^{\beta\rho\sigma\tau} \varepsilon_\rho^* (p + k)_\sigma q_\tau - \alpha \leftrightarrow \beta] h_1. \end{aligned} \quad (16)$$

Using Dirac identity

$$\sigma_{\mu\nu}\gamma^5 = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta} \quad (17)$$

in Eq. (16) one can write

$$\begin{aligned} \langle K_1(k, \varepsilon) | \bar{s} i \sigma_{\mu\nu} q^\nu b | B(p) \rangle &= \varepsilon_\mu^* [(M_B^2 - M_{K_1}^2) g_+ + s g_-] - (q \cdot \varepsilon^*) [(p+k)_\mu g_+ + q_\mu g_-] \\ &\quad + (q \cdot \varepsilon^*) [s(p+k)_\mu - (M_B^2 - M_{K_1}^2) q_\mu] h. \end{aligned} \quad (18)$$

Comparing coefficients of q_μ , ε_μ^* and $\epsilon_{\mu\nu\alpha\beta}$ from Eqs.(9), (10) and Eqs. (16) and (18), we get

$$F_1(s) = 2[g_+(s) - s h_1] \quad (19)$$

$$F_2(s) = g_+ + \frac{s}{M_B^2 - M_{K_1}^2} g_- \quad (20)$$

$$F_3(s) = -g_- - (M_B^2 - M_{K_1}^2) h. \quad (21)$$

The above results ensure that $F_1(0) = 2F_2(0)$. In terms of g_+ , g_- and h , the form factors become

$$\begin{aligned} A(s) &= \frac{M_B + M_{K_1}}{m_b - m_s} [g_+(s) - s h_1] \\ V_1(s) &= -\frac{M_B - M_{K_1}}{m_b + m_s} \left[g_+ + \frac{s}{M_B^2 - M_{K_1}^2} g_- \right] \\ V_2(s) &= -\left(\frac{M_B + M_{K_1}}{m_b + m_s} \right) [g_+(s) - s h] - \frac{2M_{K_1}}{M_B - M_{K_1}} V_0(s). \end{aligned} \quad (22)$$

By looking at the above expressions one can see that the normalization of above form factors A and V_1 at $s = 0$ is determined by the single constant $g_+(0)$ where as that of V_2 is determined by $g_+(0)$ and $V_0(s)$.

3.1 Pole contributions

The pole contribution for B to ρ has been studied in detail by Gilani *et al.*[18]. This remains the same for B to K_1 transition and again only h_1 , g_- , h and V_0 get pole contributions from $B^*(1^-)$, $B_A^*(1^+)$ and $B(0^-)$ mesons where as g_+ , g_- and $V_0(s)$ gets their contribution from quark triangle graph.

These are given by

$$\begin{aligned}
h_1|_{pole} &= -\frac{1}{2} \frac{g_{B^*BK_1}}{M_{B^*}^2} \frac{f_T^{B^*}}{1-s/M_{B^*}^2} = \frac{R_V}{M_{B^*}^2} \frac{1}{1-s/M_{B^*}^2} \\
g_-|_{pole} &= -\frac{g_{B_A^*BK_1}}{M_{B_A^*}^2} \frac{f_A^{B_A^*}}{1-s/M_{B_A^*}^2} = \frac{R_A^S}{M_{B_A^*}^2} \frac{1}{1-s/M_{B_A^*}^2} \\
h|_{pole} &= \frac{1}{2} \frac{f_{B_A^*BK_1}}{M_{B_A^*}^2} \frac{f_T^{B_A^*}}{1-s/M_{B_A^*}^2} = \frac{R_A^D}{M_{B_A^*}^2} \frac{1}{1-s/M_{B_A^*}^2} \\
V_0(s)|_{pole} &= \frac{g_{BBK_1}}{M_{K_1}} f_B \frac{s/M_B^2}{1-s/M_B^2} = R_0 \frac{s/M_B^2}{1-s/M_B^2} \quad (23)
\end{aligned}$$

where R_V , R_A^S , R_A^D and R_0 are related to the coupling constants $g_{B^*BK_1}$, $g_{B_A^*BK_1}$, $f_{B_A^*BK_1}$ and g_{BBK_1} respectively. One can find the detail about it in Ref.[18]. Thus one can write

$$A(s) = \left(\frac{M_B + M_{K_1}}{m_b - m_s} \right) \left(g_+(s) - R_V \frac{s}{M_{B^*}^2} \left(\frac{1}{1-s/M_{B^*}^2} \right) \right) \quad (24)$$

$$V_1(s) = - \left(\frac{M_B - M_{K_1}}{m_b + m_s} \right) \left(g_+(s) + \frac{s}{M_B^2 - M_{K_1}^2} \tilde{g}_- + \frac{R_A^S}{M_B^2 - M_{K_1}^2} \frac{s}{M_{B_A^*}^2} \left(\frac{1}{1-s/M_{B_A^*}^2} \right) \right) \quad (25)$$

$$V_2(s) = - \left(\frac{M_B + M_{K_1}}{m_b + m_s} \right) \left[g_+(s) - \frac{s}{M_{B_A^*}^2} R_A^D \frac{1}{1-s/M_{B_A^*}^2} \right] - \frac{2M_{K_1}}{M_B - M_{K_1}} V_0(s). \quad (26)$$

The behavior of $g_+(s)$, $\tilde{g}_-(s)$ and $V_0(s)$ near $s \rightarrow 0$ is known from LEET and their form is[18]

$$g_+(s) = \frac{\xi_\perp(0)}{(1-s/M_B^2)^2} = -\tilde{g}_-(s) \quad (27)$$

$$V_0(s) = \left(1 - \frac{M_{K_1}^2}{M_B E_{K_1}} \right) \xi_\parallel(s) + \frac{M_{K_1}}{M_B} \xi_\perp(s). \quad (28)$$

At $s \rightarrow 0$

$$V_0(0) = \frac{M_B^2 - M_{K_1}^2}{M_B^2 + M_{K_1}^2} \xi_\parallel(0) + \frac{M_{K_1}}{M_B} \xi_\perp(0) \quad (29)$$

$$E_{K_1} = \frac{M_B}{2} \left(1 - \frac{s}{M_B^2} + \frac{M_{K_1}^2}{M_B^2} \right) \quad (30)$$

$$g_+(0) = \xi_\perp(0). \quad (31)$$

The pole terms in the relations (24), (25) and (26) are expected to dominate near $s = M_{B^*}^2$ or $M_{B_A^*}^2$. On the other hand the relations obtained from Ward identities, are expected to hold for s much below the resonance region. The above behavior, near $s = 0$ and that near the pole[18] suggest

$$F(s) = \frac{F(0)}{(1 - s/M^2)(1 - s/M'^2)} \quad (32)$$

where M^2 is $M_{B^*}^2$ or $M_{B_A^*}^2$ and M' is the radial excitation of M . This parameterization not only takes into account the corrections to the single pole dominance, as suggested by dispersion relation [22, 23, 24], but also of off-mass-shell-ness of couplings of B^* or B_A^* with BK_1 channel.

Since $g_+(s)$ and $\tilde{g}_-(s)$ have no pole at $s = M_{B^*}^2$, therefore we get

$$A(s) \left(1 - \frac{s}{M_{B^*}^2}\right) \Big|_{s=M_{B^*}^2} = R_V \left(\frac{M_B + M_{K_1}}{m_b - m_s}\right)$$

This gives using the parametrization (32)

$$R_V \equiv -\frac{1}{2}g_{B^*BK_1}f_T^{B^*} = -\frac{1}{2}g_{B^*BK_1}f_B = -\frac{g_+(0)}{1 - M_{B^*}^2/M_{B^*}^2} \quad (33)$$

Similarly,

$$R_A^D \equiv \frac{1}{2}f_{B_A^*BK_1}f_T^{B_A^*} = -\frac{g_+(0)}{1 - M_{B_A^*}^2/M_{B_A^*}^2}. \quad (34)$$

For the detailed derivation and discussion on these relations we will refer to [18]. We cannot use the parametrization given in Eq.(32) for $V_1(s)$ since near $s = 0$, $V_1(s)$ behaves as $g_+(s) [1 - s/(M_B^2 - M_{K_1}^2)]$ [c.f. Eqs. (25) and (27)]. This suggests the following

$$V_1(s) = \frac{g_+(0)}{\left(1 - s/M_{B^*}^2\right)\left(1 - s/M_{B_A^*}^2\right)} \left(1 - \frac{s}{M_B^2 - M_{K_1}^2}\right). \quad (35)$$

Until now we have expressed every thing in terms of $g_+(0)$ which is the only unknown in the calculation. After the first announcement of Belle[15] for the decay $B \rightarrow K_1\gamma$, the value of $g_+(0)$ has been extracted to be [16, 17]

$$g_+(0) = \xi_\perp(0) = 0.32 \pm 0.03. \quad (36)$$

Using $f_B = 180$ MeV we have the prediction from Eq. (33)

$$g_{B^*BK_1} = 15.42 \text{ GeV}^{-1} \quad (37)$$

Similarly, the S and D wave couplings are predicted to be

$$g_{B_A^*BK_1} = 3.17 f_{B_A^*BK_1} \text{ GeV}^2 \quad (38)$$

The different values of $F(0)$'s are

$$A(0) = \left(\frac{M_B + M_{K_1}}{m_b - m_s} \right) g_+(0) \quad (39)$$

$$V_1(0) = - \left(\frac{M_B - M_{K_1}}{m_b + m_s} \right) g_+(0) \quad (40)$$

$$V_2(0) = - \left(\frac{M_B + M_{K_1}}{m_b + m_s} \right) g_+(0) - \frac{2M_{K_1}}{M_B - M_{K_1}} V_0(0) \quad (41)$$

where $g_+(0)$ is the same as given in Eq. (36). The calculation of numerical values of $A(0)$, $V_1(0)$ is very trivial but to go for $V_2(0)$, we have to know the value of $V_0(0)$. Although LEET does not give any relationship between $\xi_{\parallel}(0)$ and $\xi_{\perp}(0)$ but due to some numerical coincidence in the LCSR expressions for $\xi_{\parallel}(0)$ and $\xi_{\perp}(0)$ [25]

$$\xi_{\parallel}(0) \simeq \xi_{\perp}(0) = g_+(0) \quad (42)$$

so that, from Eq. (29)

$$V_0(0) = 1.13 g_+(0). \quad (43)$$

Thus the final expressions of form factors which we shall use for numerical work are

$$\begin{aligned} A(s) &= \frac{A(0)}{(1 - s/M_B^2)(1 - s/M_B'^2)} \\ V_1(s) &= - \frac{V_1(0)}{\left(1 - s/M_{B_A^*}^2\right) \left(1 - s/M_{B_A'}^2\right)} \left(1 - \frac{s}{M_B^2 - M_{K_1}^2}\right) \\ V_2(s) &= - \frac{\tilde{V}_2(0)}{\left(1 - s/M_{B_A^*}^2\right) \left(1 - s/M_{B_A'}^2\right)} - \frac{2M_{K_1}}{M_B - M_{K_1}} \frac{V_0(0)}{(1 - s/M_B^2)(1 - s/M_B'^2)} \end{aligned} \quad (44)$$

where

$$\begin{aligned} A(0) &= (0.52 \pm 0.05) \\ V_1(0) &= -(0.24 \pm 0.02) \\ \tilde{V}_2(0) &= -(0.39 \pm 0.03) \end{aligned} \quad (45)$$

4 Decay Distribution and Forward-Backward Asymmetry

In this section we define the decay rate distribution which we shall use for the phenomenological analysis. Following the notation from ref.[10] we can write from Eq. (3)

$$\mathcal{M} = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* m_B [\mathcal{T}_\mu^1 (\bar{l} \gamma^\mu l) + \mathcal{T}_\mu^2 (\bar{l} \gamma^\mu \gamma^5 l)] \quad (46)$$

where

$$\mathcal{T}_\mu^1 = A(\hat{s}) \varepsilon_{\mu\rho\alpha\beta} \epsilon^{*\rho} \hat{p}_B^\alpha \hat{p}_{K_1}^\beta - iB(\hat{s}) \epsilon_\mu^* + iC(\hat{s}) (\epsilon^* \cdot \hat{p}_B) \hat{p}_{h\mu} + iD(\hat{s}) (\epsilon^* \cdot \hat{p}_B) \hat{q}_\mu \quad (47)$$

$$\mathcal{T}_\mu^2 = E(\hat{s}) \varepsilon_{\mu\rho\alpha\beta} \epsilon^{*\rho} \hat{p}_B^\alpha \hat{p}_{K_1}^\beta - iF(\hat{s}) \epsilon_\mu^* + iG(\hat{s}) (\epsilon^* \cdot \hat{p}_B) \hat{p}_{h\mu} + iH(\hat{s}) (\epsilon^* \cdot \hat{p}_B) \hat{q}_\mu \quad (48)$$

The definition of different momenta involved are defined in reference[10], where the auxiliary functions are

$$\begin{aligned} A(\hat{s}) &= -\frac{2A(\hat{s})}{1 + \hat{M}_{K_1}} C_9^{eff}(\hat{s}) + \frac{2\hat{m}_b}{\hat{s}} C_7^{eff} F_1(\hat{s}) \\ B(\hat{s}) &= \left(1 + \hat{M}_{K_1}\right) \left[C_9^{eff}(\hat{s}) V_1(\hat{s}) + \frac{2\hat{m}_b}{\hat{s}} C_7^{eff} \left(1 - \hat{M}_{K_1}\right) \right] \\ C(\hat{s}) &= \frac{1}{\left(1 - \hat{M}_{K_1}^2\right)} \left\{ C_9^{eff}(\hat{s}) V_2(\hat{s}) + 2\hat{m}_b C_7^{eff} \left[F_3(\hat{s}) + \frac{1 - \hat{M}_{K_1}^2}{\hat{s}} F_2(\hat{s}) \right] \right\} \\ D(\hat{s}) &= \frac{1}{\hat{s}} \left[\left(C_9^{eff}(\hat{s}) (1 + \hat{M}_{K_1}) V_1(\hat{s}) - (1 - \hat{M}_{K_1}) V_2(\hat{s}) - 2\hat{M}_{K_1} V_0(\hat{s}) \right) \right. \\ &\quad \left. - 2\hat{m}_b C_7^{eff} F_3(\hat{s}) \right] \\ E(\hat{s}) &= -\frac{2A(\hat{s})}{1 + \hat{M}_{K_1}} C_{10} \\ F(\hat{s}) &= \left(1 + \hat{M}_{K_1}\right) C_{10} V_1(\hat{s}) \\ G(\hat{s}) &= \frac{1}{1 + \hat{M}_{K_1}} C_{10} V_2(\hat{s}) \\ H(\hat{s}) &= \frac{1}{\hat{s}} \left[C_{10}(\hat{s}) (1 + \hat{M}_{K_1}) V_1(\hat{s}) - (1 - \hat{M}_{K_1}) V_2(\hat{s}) - 2\hat{M}_{K_1} V_0(\hat{s}) \right]. \quad (49) \end{aligned}$$

The differential decay rate for $B \rightarrow K^* \mu^+ \mu^-$ can be expressed in terms of these auxiliary functions in [10] and this remains the same for $B \rightarrow K_1 \mu^+ \mu^-$ with the obvious replacements. Integration on \hat{s} in the range

$$(2\hat{m}_l)^2 \leq \hat{s} \leq (1 - \hat{m}_{K_1})^2 \quad (50)$$

with $\hat{m}_l = m_l/m_B$, and using $\tau_{B^0} = (1.530 \pm 0.009) \times 10^{-12} s$, the branching ratio is

$$\mathcal{B}(B \rightarrow K_1 \mu^+ \mu^-) = 0.9^{+0.11}_{-0.14} \times 10^{-7}$$

The above value of branching ratio is for the case if we do not include $Y(\hat{s})$ in Eq. (4). The error in the value reflects the uncertainty from the form factors, and due to the variation of input parameters like CKM matrix elements, decay constant of B meson and masses as defined in Table I.

Table I : Default value of input parameters used in the calculation

m_W	80.41 GeV
m_Z	91.1867 GeV
$\sin^2 \theta_W$	0.2233
m_c	1.4 GeV
$m_{b,pole}$	4.8 ± 0.2 GeV
m_t	173.8 ± 5.0 GeV
$\alpha_s(m_Z)$	0.119 ± 0.0058
f_B	(200 ± 30) MeV
$ V_{ts}^* V_{tb} $	0.0385

Now if we include the value of $Y(\hat{s})$ the central value of branching ratio reduces to

$$\mathcal{B}(B \rightarrow K_1 \mu^+ \mu^-) = 0.72 \times 10^{-7}$$

By including $Y(\hat{s})$ the behavior of the differential decay rate as a function of \hat{s} is shown in Fig. 1. The solid line denotes the theoretical prediction with input parameters taken at their central values, while the band between two dashed line shows the uncertainty from input parameters. In our numerical analysis we have considered only the final state leptons as being the muon. Our reason for choosing this is due to the extreme difficulty in detecting electron in the

final state and that the branching ratio $B \rightarrow K_1 \ell^+ \ell^-$ becoming small with the SM for the τ in the final state.

The differential forward-backward asymmetry for $B \rightarrow K_1 \mu^+ \mu^-$ reads as follows[10]

$$\frac{d\mathcal{A}_{\text{FB}}}{d\hat{s}} = \frac{G_F^2 \alpha^2 m_B^5}{2^{10} \pi^5} |V_{ts}^* V_{tb}|^2 \hat{s} \hat{u}(\hat{s}) [\text{Re}(BE^*) + \text{Re}(AF^*)] \quad (51)$$

where

$$\begin{aligned} \hat{u}(\hat{s}) &= \sqrt{\lambda \left(1 - 4 \frac{\hat{m}_l^2}{\hat{s}}\right)} \\ \lambda &\equiv \lambda(1, \hat{m}_{K_1}^2, \hat{s}) \\ &= 1 + \hat{m}_{K_1}^4 + \hat{s}^2 - 2\hat{s} - 2\hat{m}_{K_1}^2(1 + \hat{s}) \end{aligned} \quad (52)$$

The variable \hat{u} corresponds to θ , the angle between the momentum of the B meson and the positively charged lepton in the dilepton c.m. system frame. The behavior of forward-backward asymmetry in $B \rightarrow K_1 \mu^+ \mu^-$ decay as a function of \hat{s} is shown in Fig. 2. Contrary to the branching ratio, the forward-backward asymmetry is less sensitive to the input parameters as is clear from Fig. 2. For the zero-point of forward-backward asymmetry in the standard model, we get $\hat{s} = (0.16 \pm 0.01)$ ($s = (4.46 \pm 0.27) \text{ GeV}^2$).

Conclusions

We have studied $B \rightarrow K_1 \ell^+ \ell^-$ decay using Ward identities. The form factors have been calculated and found that their normalization is essentially determined by single constant $g_+(0)$ which has the value $g_+(0) = 0.32 \pm 0.03$ obtained from [16, 17]. By considering the radial excitation of M (where $M = M_{B^*}$ or $M_{B_A^*}$), which are suggested by dispersion relation[18], we have predicted the coupling of B^* or B_A^* with BK_1 channel as indicated in Eq. (37) and the value is $g_{B^*BK_1} = 15.42 \text{ GeV}^{-1}$. Also we have predicted the relationship between S and D wave couplings $g_{B_A^*BK_1} = 3.17 f_{B_A^*BK_1} \text{ GeV}^2$ given in Eq. (38). We have summarized our form factors in Eq. (44) and their value at $s = 0$ in Eq. (45). By using these form factors we have calculated the branching ratio for $B \rightarrow K_1 \mu^+ \mu^-$ both by considering the non resonant and resonant value of the Wilson coefficient $C_9^{eff}(\hat{s})$ which will be seen in future experiments. The decay distribution is shown graphically in Fig. 1, where the differential decay rate is plotted as a function \hat{s} .

A detailed analysis of the forward-backward asymmetry is also presented here. We have plotted the forward-backward asymmetry as a function of \hat{s}

in Fig. 2. It is clear from the graph that the the SM the central value of the zero of the FB asymmetry is at $\hat{s} = 0.16$ ($s = 4.46$). This value of the zero of the forward-backward asymmetry will provide the precision test of SM in planned future experiments.

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Figure Captions

1): The differential decay rate as a function of \hat{s} is plotted using the form factors calculated by using Ward Identities. The resonant $c\bar{c}$ states are parameterized as in refs.[19, 20]. Here the solid line denotes the theoretical predictions with the input parameters taken at their central values, while the dashed (dotted) line is for max. (min) value of input parameters.

2): The forward-backward (FB) asymmetry as a function of \hat{s} is plotted using the form factors calculated by using Ward Identities. The resonant $c\bar{c}$ states are parameterized as in refs.[19, 20]. The dashed (solid) line is for the central (max.) value of the input parameters.